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## HELAC-NLO

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## Abstract

Based on the OPP technique and the HELAC framework, HELAC-1LOOP is a program capable to numerically evaluate QCD virtual corrections to scattering amplitudes. A detailed presentation of the algorithm, along with instructions to run the code and benchmark results are given. The program is part of the HELAC-NLO framework that allows for a complete evaluation of QCD NLO corrections.

*Keywords:* QCD; NLO corrections; Scattering Amplitudes.

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## PROGRAM SUMMARY

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*Manuscript Title:* HELAC-NLO

*Authors:* G. Bevilacqua, M. Czakon, M.V. Garzelli, A. van Hameren, A. Kardos, C.G. Papadopoulos, R. Pittau, M. Worek

*Program Title:* HELAC-1LOOP

*Journal Reference:*

*Catalogue identifier:*

*Licensing provisions:* none

*Programming language:* Fortran (gfortran<sup>1</sup>, lahey95<sup>2</sup>, ifort<sup>3</sup>)

*Operating system:* Linux, Unix, Mac OS

*Keywords:* QCD; NLO corrections; Scattering Amplitudes.

*Classification:* 11.1

*Nature of problem:*

The evaluation of virtual one-loop amplitudes for multi-particle scattering is a long-standing problem (1). In the last years the OPP reduction technique (2) opened the road for a fully numerical approach based on the evaluation of the one-loop amplitude for well-defined values of the loop momentum.

*Solution method:*

By using HELAC (3–5) and CutTools (6), HELAC-1LOOP is capable to evaluate QCD virtual corrections (7). The one-loop  $n$ -particle amplitudes are constructed as part of the  $n + 2$  tree-order ones, by using the basic recursive algorithm used in HELAC. A Les Houches Event (LHE) file is being produced combining the complete information from tree- and virtual one-loop contributions. In conjunction with real corrections, obtained with the use of HELAC-DIPOLES (8), the full NLO corrections can be computed. The program has been successfully used in many applications.

*Additional comments:*

Program obtainable from: <http://helac-phegas.web.cern.ch/helac-phegas>

*Running time:*

Depending on the number of particles and generated events from seconds to days.

## References

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<sup>1</sup><http://gcc.gnu.org/fortran/>

<sup>2</sup><http://www.lahey.com>

<sup>3</sup><http://software.intel.com>

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## 1. Introduction

With the advance of LHC experiments, more precise theoretical predictions will be indispensable. In the last years many groups have been able to compute NLO corrections for multi-particle processes a task thought almost impossible before (1–15). The *NLO revolution* (16, 17) became possible because new reduction techniques have been proposed and implemented, among them the so-called OPP technique (18–21) that allows for a fully numerical evaluation of the one-loop virtual amplitude.

In this paper we describe one of the computational frameworks emerged during the last years, namely the HELAC-NLO. It incorporates several pieces of developed software, including HELAC-PHEGAS (22–24), CutTools (20), HELAC-DIPOLES (25), OneLOop (26), that have been already public for some time and HELAC-1LOOP that is presented in this paper. This program in its current form is capable to evaluate fully numerically virtual QCD corrections to scattering amplitudes involving up to 7 particles directly attached to the loop composed by strongly interacting particles (gluons and quarks). In section 2 we will briefly describe the underlying algorithm providing information on the structure of the code that may be useful for potential developers. In the next

section 3, detailed instructions on running the code are given. Benchmark results can be found in section 4, along with a description of **HELAC-DIPOLES** in order the reader and potential user to have a complete overview of the full software. Finally in section 5 we critically review the current level of sophistication and the potential improvements.

## 2. The HELAC-1LOOP algorithm

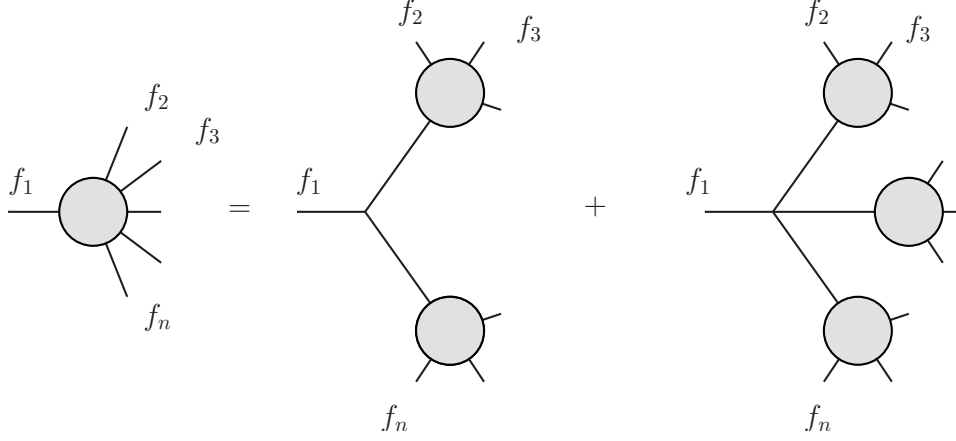
The aim of the program is to numerically evaluate the virtual contributions needed in a next-to-leading order calculation. To this end the evaluation of both tree-order and one-loop amplitudes is necessary.

The tree-order calculation algorithm is described in details in (22–24). In **HELAC-1LOOP** the first step is the construction of the so-called tree-order skeleton, that contains all information for the evaluation of the amplitude. The primary input to this construction is the flavor (`ifl(1:n)`) assignment of the  $n$  (`n`) external particles. A special file manageable by the user, also used in **HELAC**, named `constants.h`, is used to numerically fix all physical constants needed (`physics.f`).

The first action (`helac_init` in `mastef_new.f`) is to enumerate (`ncc`) and define all possible color connections (`icol(1:n,1:2)`). For a typical process consisting of  $n_q$  numbers of (outgoing)incoming (anti)quarks and  $n_g$  gluons, the number of color connections is a priori set to  $(n_q + n_g)!$ . Based on these data, the program is now constructing (within `pan1.f`) using a top-down approach, all currents<sup>4</sup> (`list(1:ngues,1:18,1:ncc)`) needed in a Dyson-Schwinger recursive representation of the amplitude, using the appropriate vertex functions (`v3` for 3-vector-boson vertex, `v4` for 4-vector-boson vertex, `vff` for vector-boson-fermion vertex, etc.), as shown schematically in the following figure

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<sup>4</sup>Notice that the number of the needed currents is not a priori known; `ngues.h` is containing a user-defined estimate of it.



Notice that if, in partitioning the particles within blobs in the above diagram we keep the order of particles untouched, this is nothing but the so-called Berends-Giele (27) recursive representation for color ordered amplitudes. At the end of this skeleton construction the set of all currents for all color connections is stored in `list`. Notice that the number of color connections (`ncc`) can now be less than its a priori defined value.

The one-loop  $n$ -particle amplitude can schematically be decomposed in a sum over terms of the form ( $m = 1, \dots, n$ )

$$\int \frac{\mu^{4-d} d^d \bar{q}}{(2\pi)^d} \frac{\bar{N}(\bar{q})}{\prod_{i=0}^{m-1} \bar{D}_i(\bar{q})} , \quad (1)$$

with  $d$ -dimensional denominators

$$\bar{D}_i(\bar{q}) = (\bar{q} + p_i)^2 - m_i^2 \quad (2)$$

where  $\bar{q}$  is the loop momentum in  $d$  dimensions and  $\bar{N}(\bar{q})$  is the numerator calculated also in  $d$  dimensions<sup>5</sup>. The sum includes of course all terms with different loop-assignment structure: two structures may differ either by the number of denominators or by the different flavor and momenta appearing in the denominators. In that sense a closed gluon, ghost or massless quark loop, for instance, with the same momentum flow, is considered as different structure, although the denominators are identical. For the highest number of denominators each loop-assignment structure (taken into account the flavor

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<sup>5</sup>When speaking about numerator function, it should be kept in mind that it generally contains propagator denominators not depending on the loop momentum

of the particles running in the loop) corresponds to a unique Feynman graph , but for  $m < n$  a collection of Feynman graphs with common loop-assignment structure should be understood.

It is a well known fact that when  $d \rightarrow 4$  limit is taken, the amplitude can be cast into the form

$$\mathcal{A} = \sum_i d_i \text{Box}_i + \sum_i c_i \text{Triangle}_i + \sum_i b_i \text{Bubble}_i + \sum_i a_i \text{Tadpole}_i + R, \quad (3)$$

where Box, Triangle, Bubble and Tadpole refer to the well known scalar one-loop functions and  $R = R_1 + R_2$  is the so-called rational term.

The reduction of Eq.(1) to Eq.(3) is the first ingredient of any approach aiming at the calculation of virtual corrections. In the following we will follow the so called *reduction at the integrand level*, developed by Ossola, Papadopoulos and Pittau (18). The main idea is that any numerator function can be written as

$$\begin{aligned} N(q) = & \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[ d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\ & + \sum_{i_0 < i_1 < i_2}^{m-1} \left[ c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ & + \sum_{i_0 < i_1}^{m-1} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\ & + \sum_{i_0}^{m-1} \left[ a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i \\ & + \tilde{P}(q) \prod_i^{m-1} D_i. \end{aligned} \quad (4)$$

where now  $N(q)$  and  $D_i(q)$  are the four-dimensional versions of  $\bar{N}(\bar{q})$  and  $\bar{D}_i(\bar{q})$ . The coefficients  $d, c, b$  and  $a$  appearing in Eq.(4) are independent of the loop momentum and the same as the ones in Eq.(3), whereas the new coefficients  $\tilde{d}, \tilde{c}, \tilde{b}, \tilde{a}$  and  $\tilde{P}(q)$ , called also spurious terms, are depending on the loop momentum and they integrate to zero.

Depending on the reduction method used, the calculation of any one-loop amplitude is placed in a very different perspective. For instance Eq.(4) can

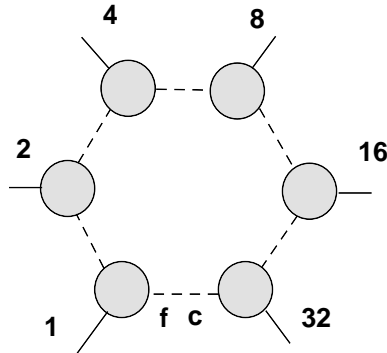
be solved by computing the numerator functions for specific values of the loop momentum, that are solutions of equations of the form

$$D_i(q) = 0, \text{ for } i = 0, \dots, M - 1 \quad (5)$$

It is customary to refer to these equations as quadruple ( $M = 4$ ), triple ( $M = 3$ ), double ( $M = 2$ ) and single ( $M = 1$ ) cuts.

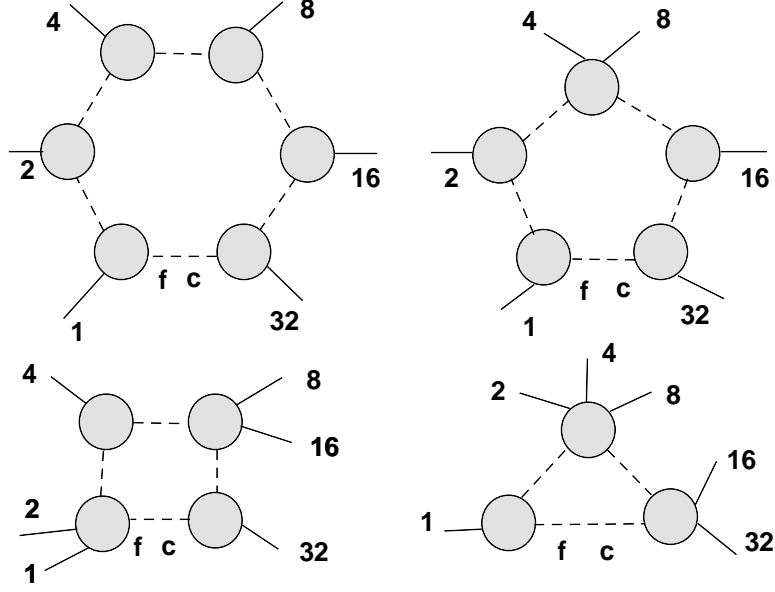
Calculating the numerator function for specific values of the loop momentum, opens the possibility to use *tree-level amplitudes* as building blocks. The reason is rather obvious: the numerator function is nothing but a sum of individual Feynman graphs with the given loop-assignment structure and as we will see in a while, it is part of a tree amplitude with  $n + 2$  particles. This is by itself a very attractive possibility, since one can use existing algorithms and tools that perform tree-order amplitude calculations, exploiting their automation, simplicity and speed. Indeed in the sequel we will describe how using **HELAC** we can also compute *any one-loop amplitude*.

For the one-loop amplitude a skeleton construction is also performed (28). For a given external configuration (**n**, **ifl**, **icol**) the construction of all topologically inequivalent partitions (i.e. permutations) of the external particles into all possible number of sets (blobs), is performed (**loop.f** and **loop/loop\_new.f90**). One such contribution is schematically represented in the following figure



For those familiar with **HELAC** the numbering of external particles follows the binary representation used. In the present example 6 particles in direct contact with the loop are considered (hexagonal topologies). The allowed particle flavors (**flavors.h**) and colors running in the loop are defined (sub-routines **check7**, **check6**, etc.): the labels **f** and **c** in the figure above refer

to the possible flavor and color of the internal loop-particles. This construction will continue to include also pentagon topologies, tetragonal topologies, triangle topologies, and bubble topologies. A typical collection of possible contributions, looks like



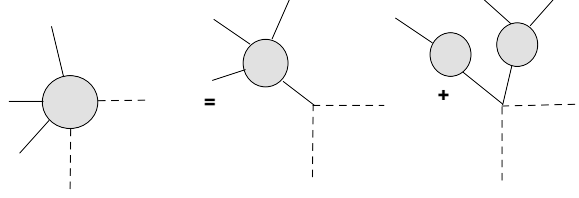
Concerning the loop-momentum flow in these constructions, the convention we have chosen is that it runs counterclockwise, and the loop-propagator connecting the blob that includes the particle number 1 and the last blob, is identified as  $\bar{D}_0(\bar{q})$  of Eq.(1).

The selection of all the above mentioned contributions is enough for the calculation of the one-loop amplitude. To help the reader to understand the concept, the construction we have followed is equivalent to draw all possible one-loop Feynman graphs, and then collect them in sub-classes that are characterized by a common loop-assignment structure (after possible momentum shifts).

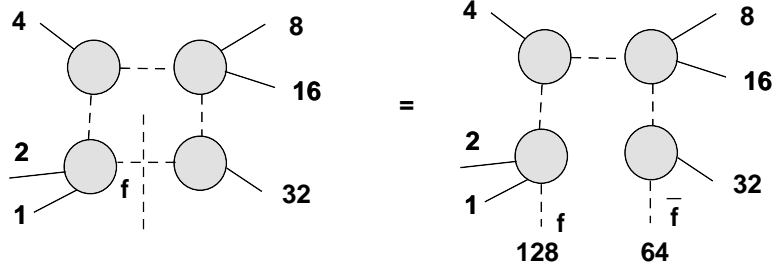
In practice now, each *numerator* contribution, will be calculated as part of the  $n + 2$  tree-order amplitude subject to the constraint that the attached



blobs will contain no propagator depending on the loop momentum



and no denominator will be used for the internal loop propagators. Cutting now the line connecting the blob containing the particle number 1 and the last blob, it is easy to see that we have nothing more than a part of the  $n + 2$  amplitude. The 'cut' particles, with flavor  $f$  and color connection (`icol`) appropriately defined (28), will now acquire their usual numbering of external particles in HELAC, namely  $2^n$  and  $2^{n+1}$ , (64 and 128 for  $n = 6$ ).



Using the above data the program knows how to reconstruct (subroutines `numX_cuttools`,  $X = 2 \dots 7$ , in `loop.f`) all information needed for the calculation and store it as a sequence of sub-amplitudes or currents (`listnum`), quite as in the tree-order calculation, the main difference being that for a given color connection we have generically more than one contributions (`nonums`) characterized by different partitions of external particles, as well as flavors and colors running in the loop.

Rational terms are classified in two categories:  $R_1$  are evaluated by `CutTools` (`cts_xcut:rat1`); the  $R_2$  rational terms are calculated through Feynman rules as established in (29–32). In `HELAC-1LOOP:physics.f` in addition to all Standard Model couplings defined as in `HELAC`, couplings related to the  $R_2$  rational terms are also defined. Although we have to deal with a tree-order construction, the skeleton is build up following a procedure similar to that of the loop amplitude. The reason is that the special  $R_2$  vertex has to appear only once for arbitrary scattering process at one loop.

Moreover these  $R_2$  vertices can join up to 4 particles. Therefore the skeleton construction (`loop.f`) starts with the distribution of all external particles in 4, 3 and 2 sub-sets (blobs). Then these blobs, that represent currents or sub-amplitudes, are recursively (`loop.f:mumX_r2` with  $X=2,3,4$ ) defined in terms of the external particles and SM vertices, using the basic **HELAC** algorithm. The numerical evaluation of these contributions is identical to that of the tree-order amplitude.

The data collected so far are used to evaluate numerically the amplitude in the so-called second stage. At this moment 4-momenta have to be supplied. In **HELAC-1LOOP** we provide three ways for dealing with 4-momenta: the testing modes (`irambo=1, iint=1`) in which case the 4-momenta are generated randomly and one can use them for a number (`nmc`) of evaluation of matrix element and (`irambo=0`) in which case the 4-momenta are provided by the user through a file (`mom`); the third way (`irambo=2, iint=0`), the so-called *re-weighting mode*, is working through the reading of a Les Houches Events (LHE) file generated by the **HELAC** itself.

Once 4-momenta are supplied, the external wave function vectors (4-dimensional complex vectors from `wave.f`) are computed. Then the tree-order amplitude for each color configuration is evaluated (`pan1.f:nextq`). The total matrix element squared is then evaluated (`master_new.f`) according to the standard formula

$$\sum_{\sigma, \sigma'} A_{\sigma}^* \mathcal{C}_{\sigma, \sigma'} A_{\sigma'} . \quad (6)$$

The color matrix  $\mathcal{C}$  (`rmatrix`) has been already computed and stored in the first stage.

For the one-loop amplitudes we follow the OPP reduction (`nextq1`). To this end the **CutTools** program is interfaced and used (`cuttools.intf.f`). The input to **CutTools** is the numerator function (`numerator`) evaluated at values of the loop momentum provided by **CutTools** itself. In order to compute the numerator function, the polarization vectors of the two extra 'external' particles, after the one-particle cut, are also calculated. Within the Feynman gauge for gauge bosons, the sum over four different (4-dimensional) polarizations, that satisfy  $\sum_i e_i^{\mu} e_i^{\nu} = g^{\mu\nu}$  is performed. Ghost particles are also included. Finally for fermions, four vectors in spinor space, satisfying  $\sum_i u_{\alpha}^{(i)} u_{\beta}^{(i)} = (\not{q} + m)_{\alpha\beta}$  are used.

In the *re-weighting mode*, the actual calculation of the virtual corrections is organized using a re-weighting technique (33, 34). To explain how this

works, let us start with the following equation ( $\mathcal{M}$  is the tree-order and  $\mathcal{L}$  the one-loop virtual matrix elements)

$$\sigma_{ab}^{LO+V} = \int dx_1 dx_2 d\Phi_m f_a(x_1) f_b(x_2) \left( |\mathcal{M}|^2 + \mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L} \right), \quad (7)$$

which gives the sum of leading order (LO) and virtual (V) contributions for a scattering  $ab \rightarrow m$ -particles. It can be re-written as

$$\sigma_{ab}^{LO+V} = \int dx_1 dx_2 d\Phi_m f_a(x_1) f_b(x_2) |\mathcal{M}|^2 \left( 1 + \frac{\mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L}}{|\mathcal{M}|^2} \right). \quad (8)$$

Since  $\mathcal{L}$  is a time consuming function one would like to calculate it as few times as possible. To this end a sample of un-weighted events is produced based on the tree order distribution, namely

$$g(\vec{X}) \equiv g(x_1, x_2, \Phi_m) = \frac{1}{\sigma^{LO}} \frac{d\sigma_{ab}^{LO}}{dx_1 dx_2 d\Phi_m}, \quad (9)$$

satisfying  $\int d\vec{X} g(\vec{X}) = 1$ . The sample  $S$  of un-weighted events has the following property,

$$\frac{1}{N_S} \sum_{i \in S} \mathcal{O}(\vec{X}_i) = \int d\vec{X} g(\vec{X}) \mathcal{O}(\vec{X}), \quad (10)$$

where the equality should be understood in the statistical sense, and  $\mathcal{O}(\vec{X})$  is any well-defined function over the integration space. Now it is trivial to see that if

$$\mathcal{O}(\vec{X}) = \left( 1 + \frac{\mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L}}{|\mathcal{M}|^2} \right), \quad (11)$$

then

$$\frac{1}{N_S} \sum_{i \in S} \mathcal{O}(\vec{X}_i) = \frac{\sigma_{ab}^{LO+V}}{\sigma_{ab}^{LO}}. \quad (12)$$

In practice the sample of tree order un-weighted events includes all information on the integration space, namely, the color assignment, the (random) helicity configuration, the fractions  $x_1$  and  $x_2$  and the  $m$ -body phase-space. For future convenience it is produced in a standard Les Houches format (35). One-loop contributions are only calculated for this sample of un-weighted events, and the weight assigned to each of those events is given by

$$w = \frac{\mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L}}{|\mathcal{M}|^2}. \quad (13)$$

The total virtual contribution can now be easily estimated by

$$\sigma^V = \langle w \rangle \sigma^B, \quad (14)$$

where  $\sigma^B$  is the Born cross section, already included in the LHE file. Moreover, the sample of events including the information on  $w$ , can be used to produce any kinematical distribution, according to the Eq. 10.

Most reduction algorithms suffer from numerical instabilities usually caused by the presence of small Gram determinants. In order to detect the phase-space points with unstable behavior with respect to the reduction procedure, one can rely on several tests (36). In the current version of **HELAC-1LOOP** we employ actually the so-called gauge test. Our experience shows that the effect of numerical instabilities is more pronounced when higher rank tensor integrals are involved (namely high powers of loop momentum in the numerator function). This is often correlated with the presence of gluons in the scattering amplitude under consideration. In that case the gauge test can be performed and it is equivalent to replace the polarization vector of an external gluon with its momentum. Events that fail to obey the Ward identity are separated from the initial sample. They can be treated then in higher (quadruple) precision. In any case experience shows that only for very complicated processes, like for instance  $gg \rightarrow t\bar{t}gg$ , the effect is appreciable. Care should be taken in properly defining the numerical criterion of rejection. For the moment we replace the gluon polarization vector with the gluon momentum normalized to its energy  $\epsilon^\mu \rightarrow p^\mu/p^0$  and reject events when the computed matrix element differs from zero by an amount greater to  $10^{-9}$ . On the other hand we should emphasize that this procedure cannot be the final answer, since it has a limited applicability depending on the presence of external gluons in the scattering process under consideration.

We should also notice that for being able to test the calculations we provide the numerical evaluation of the infrared part (**w1\_I**) of the so-called  $I$ -operator. This is achieved as described in (28) by computing the color-correlated matrix (**rmatrix\_I**) and then using the tree-order amplitudes.

### 3. How to run the code

The code is written in **Fortran 90** and needs no additional software to run. We use the fortran compiler **gfortran** as the default, but in many applications we have used also **lahey95** and **ifort**. Unpacking the distributed

tarball `helac1loop.tgz` will create the directory `HELAC1L_OFFICIAL`. In this you will find the subdirectories `examples`, `run`, `src/1LOOP`, `src/utils` and `src/TREE`. For the convenience of the potential user, three script files named `run_testing`, `run_reweight` and `run_GC` can be found in the subdirectory `run` that can guide the user to run the code as described in details below.

### 3.1. 1LOOP

- Edit the file `constants.h` and define your own physical parameters if needed (see (24) for explanation).
- Run the script file `scriptmake`  
`./scriptmake n1 n2 n3 n4` where `n1` 0 means normal double precision and 1 quadruple (if supported by fortran version), `n2` 0 for compiling everything from scratch and 1 only the latest files, `n3` is empty by default and can be used to pass additional flags to fortran commands (see the `scriptmake` file), and finally `n4` either empty or `GC` that means gauge check mode. For instance the command

```
./scriptmake 0 0 "" "GC"
```

will produce the `main_onep_dpGC.exe` executable file, for use in gauge check, where

```
./scriptmake 0 0
```

will produce the `main_onep_dp.exe` executable file.

- Edit the input file `input` and define appropriately the input parameters:
  - `iint` 0 for re-weighting mode, 1 for all others
  - `ibv` 0 for full summation over colors, 1 for Monte Carlo over colors
  - `iverbose` 0...3 different levels of verbosity
  - `repeat` 1 only skeleton construction, 2 only amplitude evaluation (assuming the skeleton is present), 0 for one shot
  - `iranhel` 0 for full summation over helicities 1 for MC over helicities
  - `n` number of particles

- **flavors** flavor of particles according to the HELAC list
- **iflag** 0 for internal definition of helicities 1 for user providing helicities
- **ihiggs** 0 if no Higgs is included 1 if Higgs is included if allowed
- **loopi** T if loop amplitude is calculated F if only tree is calculated
- **only** T if only QCD couplings are allowed, F if also EW couplings are allowed
- **withqcd** T if QCD is included, F elsewhere
- **irambo** 0 if momenta are provided by the user (through **mom** file), 1 if are generated randomly by **RAMBO**, 2 if provided by the **.lhe** LHE file generated by **HELAC**
- **e** energy in GeV
- **nmc** number of phase-space points to be evaluated (for **irambo** 0 or 1)
- **mom** file with the 4-momenta in the format  $E, p_x, p_y, p_z, m$
- **momout** output file for user provided momenta
- **muscale** the renormalization scale

### 3.2. TREE

In this directory you can generate the LHE file, for later use in the re-weighting procedure. The generation is identical to the standard **HELAC-PHEGAS** procedure. So we refer the potential user to (24). Some new elements with respect to the standard treatment have been added, without affecting the generation procedure.

- In this version a color MC is being used according to (5). There is a new keyword **color\_flag** 0 for a full color summation and 1 for a MC over colors.
- An interface to phase-space generator **KALEU** (37) is also present, the user can set the **phasespace\_flag** to 0 for **PHEGAS** or 1 for **KALEU**.
- Finally the **oneloop\_rewgt** keyword has to be set to true (T) in order the generated LHE to be used for the re-weighting procedure described above.

The tree-order generation will result to a LHE file, **sampleGOGOTqTa.lhe** for instance for the process  $gg \rightarrow t\bar{t}$ .

## 4. Results and Benchmarks

In this section we will give characteristic examples of running the code. The tarball file also contains the `examples` directory where these results are stored.

### 4.1. The testing mode

Let us choose a relatively simple example  $u\bar{d} \rightarrow W^+ + ng$  with  $n = 1, 2, 3$ . We start with simplest case  $n = 1$ . The input file looks like: The output file (the reader can find it as `uDWg_gen` in the official distribution `examples`) incorporates information on how the different currents are formatted. The main result of this run is encoded in the files

- `tree_UqDaW+G0.in` including all the information needed by HELAC-1LOOP to evaluate numerically the amplitude and
- `treeli_UqUaW+G0.in` with some brief description on the virtual amplitude generation

Now by changing the `repeat` keyword value from 1 to 2, we have the numerical evaluation of the amplitude in `uDWg_out`. Most of the content of the output file is self-explanatory. We focus here on its major aspects. After the printout of physical constant the user (if `iverbose`  $\geq 1$ ) will see the following

UqDaW+G0

a line printing out the process under consideration  $u\bar{d} \rightarrow W^+g$ . Then a bunch of lines like

```
INFO =====
INFO COLOR          1 out of          2
INFO  2   6  -3   5   1   1   4  34   3   2  -4   2   0   0   0   1   1   0
INFO  2  10  -4   6   1   1   8  35   4   2  -4   2   0   0   0   1   1   3
INFO  2  10  -4   6   0   1   8  35   4   2  -4   2   0   0   0   2   1   3
INFO  2  14  -3   7   1   2   4  34   3  10  -4   6   0   0   0   1   1   0
INFO  2  14  -3   7   2   2   8  35   4   6  -3   5   0   0   0   1   1   3
INFO  2  14  -3   7   0   2   8  35   4   6  -3   5   0   0   0   2   1   3
```

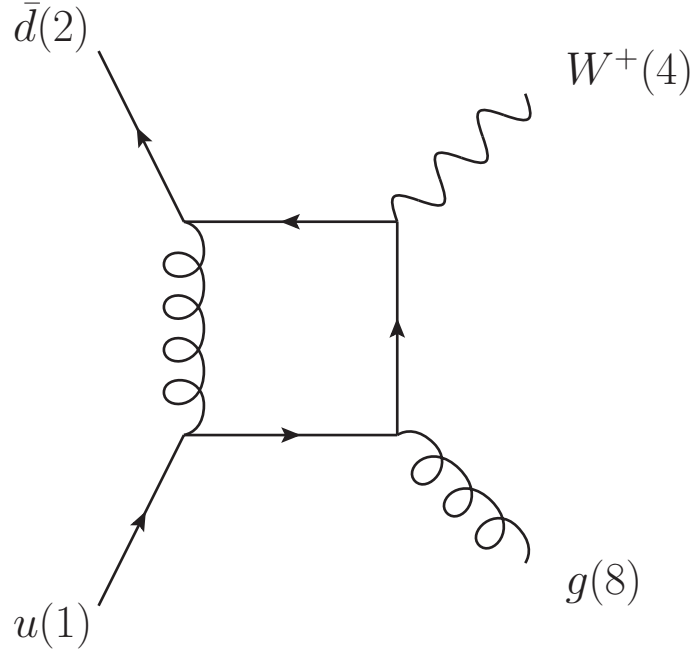
where each line represents a sub-amplitude needed in the construction of the tree-order amplitudes corresponding to the 1 out of 2 color connection configurations existing for this process. Then after exhausting all tree-order information you start seeing

```

LOOP T
INFO =====
INFO COLOR          1 out of          2
INFO number of nums          33
INFO NUM            1 of          33          6
INFO  3  24   3   7   1   1   8  35   4  16   3   5   0   0   0   1   1   3
INFO  3  24   3   7   0   1   8  35   4  16   3   5   0   0   0   2   1   3
INFO  3  28   4   8   1   1   4  34   3  24   3   7   0   0   0   1   1   0
INFO  1  30  35   9   1   1   2  -4   2  28   4   8   0   0   0   0   1   2
INFO  2  62  -3  10   1   1  30  35   9  32  -3   6   0   0   0   1   1   2
INFO  2  62  -3  10   0   1  30  35   9  32  -3   6   0   0   0   2   1   2
INFO  4   8   4   2   1   3   3   4  35   0   0   0   0   0   0   0   3   1
INFOYY  1

```

again for the 1st color connection. As it is printed there are now 33 (number of nums 33) contributions and the first one is nothing but the following box graph



The relevant topological information can easily be read off from the line

```

INFO  4   8   4   2   1   3   3   4  35   0   0   0   0   0   0   0   3   1

```

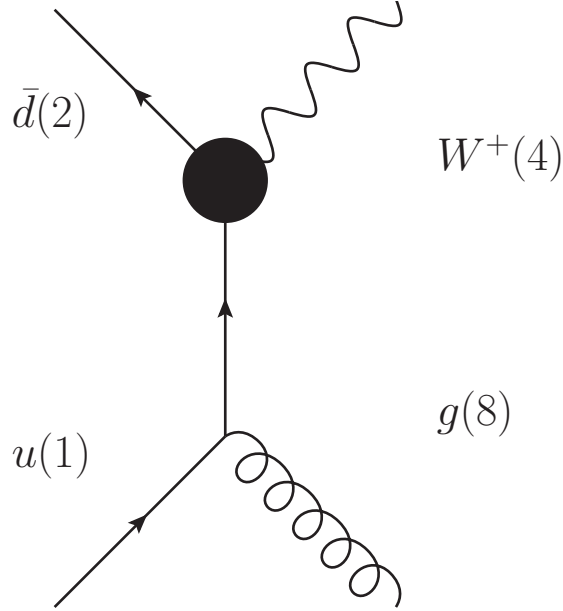
the first number after the keyword INFO, namely 4, being the number of loop propagators, then 8 4 2 1 the ordering of blobs and finally 3 3 4 35 ( $udg$ ) the corresponding flavor assignment.



Out of the 33 contributions, the last 6 refer to  $R_2$  tree-order like ones including a unique special vertex each. In the following example a special  $R_2$   $u\bar{d}W$  vertex

INFO	NUM			29	of		33		3									
INFO	25	6	-3	5	1	1	4	34	3	2	-4	2	0	0	0	1	1	0
INFO	2	14	-3	6	1	1	8	35	4	6	-3	5	0	0	0	1	1	3
INFO	2	14	-3	6	0	1	8	35	4	6	-3	5	0	0	0	2	1	3
INFO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
INFOYY	1																	

is used as shown also schematically in the following Feynman graph:



At the end of the file, after momenta have been generated and printed, you find the most important result which is the numerical values of matrix elements:

```

HELICITY CFGS          6
total amplitude squared L0   =   0.73261294162118751
total amplitude squared U0 =   4.88872424257442914E-002
total amplitude squared T0 =   4.88872424257442914E-002
ratio =   6.67299738352457872E-002
total amplitude squared U1 =   2.38831983483179001E-002
total amplitude squared T1 =  -2.26466317876591848E-002
total amplitude squared T2 =  -6.87832271575313514E-002
total amplitude squared I1 =  -2.26466317876590842E-002

```

```

total amplitude squared I2 = -6.87832271575313653E-002
total amplitude squared R0 =  0.00000000000000000
total amplitude squared R1 = -4.65298301359770849E-002

```

In this case there are 6 non-zero helicity configurations, L0 is the leading order  $|\mathcal{M}|^2$  properly summed and averaged, U0 is the finite part of the one-loop amplitude not including coupling constant renormalization (but including mass renormalization if present), T0 the renormalized one, and so on for U1, T1 for the  $\frac{1}{\epsilon}$  and U2, T2 for the  $\frac{1}{\epsilon^2}$  terms. The I1 and I2 are the corresponding poles predicted by the  $I$ -operator and the level of agreement between T2 and I2 as well as between T1 and I1 reflect the precision achieved.

In the website of HELAC-NLO, results can also be found for  $u\bar{d} \rightarrow W^+gg$  and  $u\bar{d} \rightarrow W^+ggg$ .

#### 4.2. Re-weighting mode

The distributed version contains reproducible results for the process  $gg \rightarrow t\bar{t}$ . To obtain those results you have to first generate in **TREE** sub-directory the LHE file by just executing the following command

```
./run.sh user.inp myenv
```

as in the usual HELAC-PHEGAS. In general the user has to edit the **user.inp** input file and define the corresponding input parameters.

Copy the produced sample file **sampleGOGOTqTa.lhe** in the same directory where the **main\_onep\_dpGC.exe** is used in case you have to run a gauge test. To this end the set-up of the input parameters has to be as follows: **iint=0**, **ibv=1** and **irambo=2**. When now the executable is running two sample LHE files will be generated **sampleGOGOTqTa\_GAUGECHECK\_FAILED.lhe** and **sampleGOGOTqTa\_GAUGECHECK\_PASSED.lhe**, containing the events that have failed and passed the test respectively.

Then rename this last file to **sampleGOGOTqTa.lhe** and run

```
./main_onep_dp.exe < input ><<your output>>
```

The final result is the LHE file **sampleGOGOTqTa\_WEIGHTED.lhe** which can now be used in conjunction with the HELAC-DIPOLES to get the full NLO corrections.

The events if any, in `sampleGOGOTqTa_GAUGECHECK_FAILED.lhe` can be reprocessed in higher numerical precision<sup>6</sup>:

```
./main_onep_qp.exe < input ><<your output>>
```

The resulting file is again `sampleGOGOTqTa_WEIGHTED.lhe` which can now be combined<sup>7</sup> with the one produced with the passed events. Care of course has to be paid by the user in properly managing and re-naming the emerging files.

#### 4.3. *HELAC-DIPOLES*

In this subsection we will briefly review the use of **HELAC-DIPOLES** in order the user to be able to have a more complete overview of the full software. For a detailed description please refer to (25).

Unpack the tarball `dipoles.tgz` inside the main **HELAC** directory and copy `alphas_std.h` from the `dipoles` directory to **HELAC**. This sets up two-loop running consistently with the PDFs used by **HELAC-DIPOLES**.

- Configure **HELAC-PHEGAS**
  - Edit file `myenv` to setup the libraries and compilers,
  - Edit `user.inp`, to specify the input physical parameters,
  - Edit `getqcdscale.h`, if a running coupling constant is needed,
  - Edit `nh.h`, to specify the number of histograms that will be generated.

Apart from `myenv`, for all other files described so far the user may use their default configuration. To run

```
./run.sh user.inp myenv
```

This procedure will set-up several parameters needed by **HELAC-DIPOLES** later on, among others the cuts to be used by the integrated dipoles (to be run with `make run_I` and `make run_KP`, see below).

---

<sup>6</sup>Such a possibility can be realized with `lahey95` and `ifort` that incorporate at the compilation level the option of quadruple precision. See also the script file `scriptmake` for more details.

<sup>7</sup>In the sub-directory `src/Utils/COMBINE` useful tools for combining LHE files are available.

- Configure HELAC-DIPOLES
  - Edit `dipoles.input`, to specify the process and various optimization parameters just as in the original HELAC,
  - Edit `dipoles.conf`, to set specific parameters for the calculation of the dipoles, as follows:
    - \* `onlyreal`: if set to true, only real corrections for a given process will be calculated. The cuts are then specified in `cuts.h`. The result must coincide with the one of the original HELAC with the same input parameters. This option is included for testing purposes.
    - \* `onlylast`: if set to true, only those dipoles will be included, which contain the last particle (for correctness it must be a parton). This is useful for some processes, where it is clear that only the last particle can be soft/collinear, and the book-keeping remains simple to obtain the full result at NLO.
    - \* `onlydiv`: if set to true, only divergent dipoles will be included. Non-divergent dipoles correspond to a pair of massive quarks in the final state. They are only useful to get rid of large Sudakov logarithms, but are not essential for the finiteness of the real radiation contribution.
    - \* `hybrid`: if set to true, non-parton polarizations will be summed over by a continuous Monte Carlo integration over a phase parameter.
    - \* `signmode`: defines how positive and negative contributions are to be treated: 0 - the result is left unchanged, whether positive or negative, 1 - only positive numbers, a negative result is set to zero, 2 - only positive numbers, but the sign of a negative result is changed, and positive results are set to zero
    - \* `sumtype`: in the first phase (preferably for phase space optimization), the summation over helicities of the partons can be performed in three different ways: 0 - exact fast summation (independently for real radiation and dipoles), 1 - exact slow summation (for a given helicity configuration both real radiation and the dipole sum will be calculated), 2 - flat

Monte Carlo summation over all non-vanishing helicity configurations. In practice, it is recommended to use the last option.

- \* **nsumpol**: number of accepted points to be summed over helicity with the method specified by **sumtype**. The counting starts after phase space optimization is finished.
- \* **noptpol**: number of accepted points to be used for helicity sampling optimization. During helicity sampling optimization, slow summation over helicity configurations (in the sense defined in the description of **sumtype**) is performed. It is therefore recommended to keep this number relatively small (of the order of a few hundred to a thousand).
- \* **nuptpol**: number of accepted points after which an updated of the helicity sampling weights is performed. This number should be rather large for best results (at least an order of magnitude larger than **noptpol**).
- \* **alphaMinCut**: lowest value of **alphaMin**, below which a point will be rejected altogether, because of risk of numerical instabilities. For exact definition of **alpha**, see (25).
- \* **alphaMaxII**, **alphaMaxIF**, **alphaMaxFI**, **alphaMaxFF**, **kappa**: parameters of the dipoles (see (25)).
- **jetfunctions.f**, for non-trivial jet functions, although most work should be performed on **FJmpo** (jet function for real radiation),
- **cuts.h**, to specify general cuts to be used by the jet functions,
- **histograms.f**, to define histograms,
- **seed.input**, to change the random number seed, which is useful for trivial parallelization runs (this should be an integer).

Again one can use the existing default configuration. To run the program

```
export FC="<<your fortran>>"
make
make run
```

This will compile the dipole subtracted version and run it. In case this is the first run for a given process, the calculation will be stopped and you should run

```
make trees
make run
```

The former will generate and store in the subdirectory `helac_trees` all skeleton files for the different subprocesses needed by the dipole-subtraction.

- For the  $I$  operator the corresponding input and configuration files are `dipoles_I.input` and `dipoles_I.conf`. The parameters defined in the configuration file are: the renormalization scale, the number of light  $N_f$  and heavy  $N_F$  quark flavors, used in the definition of  $I$  operator (25, 38), the  $\alpha$  parameter that controls the integration of dipole functions over the available phase-space and the parameter  $\kappa$  defined in (38). To run

```
make run_I
```

- For the  $K + P$  operator the corresponding input and configuration files are `dipoles_KP.input` and `dipoles_KP.conf`. In addition to the common parameters described so far for the  $I$  operator, the configuration file contains also the definition of the factorization scale and the number and flavors of initial state partons to be taken into account. To run

```
make run_KP
```

The result of the run consists of several files. A typical output will have at the end the following information

```
out of      1000000      1000001 points have been used
and        340079 points resulted to != 0 weight
whereas      659922 points to 0 weight
estimator x:   0.381485D-16
estimator y:   0.143375D-32
estimator z:   0.205540D-65
average estimate :   0.381485D-16
                  +\ -   0.378649D-16
variance estimate:  0.143375D-32
                  +\ -   0.143367D-32
be aware that the error estimate may be bad!
estimator x:    0.835522D-04
```

```

estimator y:    0.156118D-12
estimator z:    0.930801D-29
average estimate : 0.835522D-04
                +\ - 0.395118D-06
variance estimate: 0.156118D-12
                +\ - 0.305090D-14
total XS -8.35522401509068940E-005  3.95117621007448708E-007
lwri: points have used 0.0000000000000000
      2212      2212  7000.00000000000000  7000.00000000000000
% error: 99.256586045324298
% error: 0.47289889570121785

```

3

which states that from a run of 1 million points, 340,079 have been used after cuts and the positive part of the cross section is 0.381485D-16 with a statistical uncertainty +\ - 0.378649D-16 whereas the negative part is 0.835522D-04, +\ - 0.395118D-06 (in nanobarns). See also (24) for more explanations. Also files named `hi_file`, `hi_file_I`, `hi_file_KP` will be generated with all data needed for the histograms.

## 5. Outlook

The progress in NLO calculations seen over the last years has made the development of an automatized computational framework a realistic task (2, 20, 25, 28, 39–42). In as much as in the tree-order generation NLO programs will be able to generate LHE files ready for use in physics analyses. Nevertheless there are several open issues to be addressed in the near future:

- For the moment LHE can be generated for tree-order plus virtual corrections. For real corrections usually one has to deal with so-called "weighted" events, namely a large collection of phase-space integration points. Taking into account also that the Monte-Carlo convergence of the real corrections, especially the real-subtracted part is quite slow, a solution will be very welcome. Within `HELAC-NLO` we plan to further investigate the possibility of using alternative phase-space algorithms, sampling over colors and other subtraction methods in order to achieve a significant improvement in the overall efficiency.
- The interface to Parton-Shower (43–46) programs is also an important issue at the phenomenological level. There are already several steps taken towards this direction. So far, `HELAC-1LOOP` has been interfaced to the `POWHEG-BOX` framework at the purpose of studying specific processes like  $pp \rightarrow ttH$  (47) and  $pp \rightarrow ttj$  (48), including NLO

QCD corrections matched to a Parton Shower evolution, followed by hadronization and hadron decay, up to final predictions at the hadron level to be compared to LHC and Tevatron data. We plan to further investigate this issue with the aim to integrate and automatize the full procedure.

- To address the incorporation of the full set of Electroweak corrections as well as theories beyond the SM, a re-implementation of the **HELAC** algorithm is desirable. In such a process, we aim also to include several straightforward improvements in constraining the redundancy of the actual computation, resulting to a significant reduction in computing time and resources.

Finally it will be very interesting to advance beyond one loop. Both the OPP reduction method and the recursive approach to scattering amplitude computation may open the road to highly efficient calculations at the two-loop level (49).

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